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An approximate initial value treatment of the neutron slowing down equation

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A method for approximately determining the coefficients in the series solution of the slowing down equation with isotropic elastic scattering and constant cross sections is described

1. Introduction

An exact treatment of the neutron slowing down equation (isotropic elastic scattering, constant cross sections) involves the evalution of the real and imaginary roots of the transcendental equation

$$\xi^* = 1-\left(\frac{\xi^*}{a}-1\right)\alpha\frac{\exp(a\epsilon/\xi^*)-1}{1-\alpha\exp(a\epsilon/\xi^*)} \quad \ldots (1)$$

Then the collision density $F(u)$ is given by

$$F(u) = \sum_{n=0}^{\infty} \bar{b}_n(\xi_n^*)\exp(-au/\xi_n^*), \quad \ldots (2)$$

where the $\bar{b}_n(\xi_n^*)$ are obtained from Laplace transform methods, (Sengupta & Srikantiah 1974) This is an exact solution of the slowing down equation as the ξ_n^* and $\bar{b}_n$ are obtained exactly In this short paper, we determine the $\bar{b}_n$ approximately by an initial value treatment of the slowing down equation.

2 The Initial Value Method

Consider the equation to the slowing down density

$$q(u) = \lambda s\int_{u-\epsilon}^{u} du'(\exp(u'-u)-\alpha)F(u'), u > \epsilon, \quad \ldots (3)$$

where $F(u)$ and $q(u)$ are the collision and slowing down densities respectively, and the other notations are as given by Sengupta & Srikantiah (1974) Introducing

$$b_m(\xi_m^*) = \xi_m^*\bar{b}_m(\xi_m^*)$$

in eq. (2) and using it in eq. (3) gives

$$q(u) = \sum_{m=0}^{\infty} b_m(\xi_m^*)\exp(-au/\xi_m^*). \quad \ldots (4)$$

Let

$$q_m(u) = b_m(\xi_m^*) \exp(-au/\xi_m^*),$$

and

$$\underset{m}{F}(u) = \frac{b_m(\xi_m^*)}{\xi_m^*} \exp(-au/\xi_m^*),$$

then

$$q(u) = \Sigma\ q_m(u), \quad \ldots \quad (5a)$$

$$F(u) = \Sigma_m F(u), \quad \ldots \quad (5b)$$

and

$$q_m(u) = \xi_m^* F_m(u), \quad \ldots \quad (6)$$

that is each of the roots ξ_m^* is given as the ratios of the respective densities.

To obtain b_m we note that

$$q(\epsilon) = \sum_{m=0} b_m \exp(-a\epsilon/\xi_m^*)$$

that is

$$q_m(\epsilon) = b_m \exp(-a\epsilon/\xi_m^*)$$

and the equation to the slowing down density becomes

$$q(u) = \sum_{m=0} q_m(\epsilon) \exp(-a(u-\epsilon)/\xi_m^*) \quad (7)$$

Also note that

$$F(u) = \sum_{m=0}^{\infty} \frac{q_m(\epsilon)}{\xi_m^*} \exp(-a(u-\epsilon)/\xi_m^*) \quad (8)$$

To obtain the $q_m(\epsilon)$, we employ the technique used in the approximation scheme developed previously (Sengupta 1973), that is we differentiate eq (7) :

$$\frac{d^n q}{du^n} = \sum_{m=0}^{\infty} (-)_n (a/\xi_m^*)^n\ q_m(\epsilon) \exp(-a(u-\epsilon)/\xi_m^*),\ n = 0, 1, 2, \ldots \quad (9)$$

and use the initial values of $q^{(n)}(u)$ at $u = \epsilon$. This gives the q_m as the solution to the following infinite system of equations

$$\begin{aligned} q_0+q_1+q_2+\ldots &= q(\epsilon), \\ z_0q_0+z_1q_1+z_2q_2+\ldots &= -q'(\epsilon), \\ z_0^2q_0+z_1^2q_1+z_2^2q_2+\ldots &= q''(\epsilon), \end{aligned} \quad \ldots \quad (10)$$

where

$$z_m = (a/\xi_m^*),$$

Now in the interval $\epsilon < u < 2\epsilon$, $F(u)$ is given by

$$F(u) = \lambda s \exp((-1-\lambda s)u)(1-\exp(-\lambda \epsilon s)(1+\lambda s(u-\epsilon))),$$

Therefore,

$$\frac{d^nF}{du^n} = -(1-\lambda s)\frac{d^{n-1}F}{du^{n-1}} + (-)^n(\lambda s)^2(1-\lambda s)^{n-1}\exp(-\lambda\epsilon s)\exp(-(1-\lambda s)u),$$

and

$$\frac{d^nF}{du^n}(\epsilon_+) = (-)^n(\lambda s)^2\alpha(1-\lambda s)^{n-1} - (1-\lambda s)\frac{d^{n-1}F}{du^{n-1}}(\epsilon_+)$$

from which the following relation for the derivatives at ϵ can be easily obtained

$$\frac{d^nF}{du^n}(\epsilon_+) = (-)^n(1-\lambda s)^{n-1}(n\alpha(\lambda s)^2+(1-\lambda s)F(\epsilon_+)). \qquad \dots \quad (11)$$

Using this in the condition

$$\frac{d^nq}{du^n} = -a\frac{d^{n-1}F}{du^{n-1}},$$

eqs (8) and (9), gives the required expression for d^nq/du^n at ϵ_+

Now since the ξ_m'*, $m = 1, 2, 3, \dots$ are complex, the $q_m, m = 0, 1, 2, \dots$ will also be complex Let

$$q_m = a_m + ib_m$$

$$(\lambda^*a)_m = c_m + id_m, \; (\lambda^*a)_0 = c_0, \; (\lambda^*a)_m = -\frac{a}{\xi_m^*}$$

then quantities of the type $(a_m+ib_m)(c_m+id_m)^n$ arise in the solution of eq. (10) Noting that

$$\begin{aligned}(a_m+ib_m)(c_m+id_m)^n &= a_m(c_m^n - {}^nC_2c_m^{n-2}d_m^2 + {}^nC_4c_m^{n-4}d_m^4 - \dots)\\ &\quad -b_m(nc_m^{n-1}d_m - {}^nC_3c_m^{n-3}d_m^3 + {}^nC_5c_m^{n-5}d_m^5 - \dots)\\ &\quad +ib_m(c_m^n - {}^nC_2c_m^{n-2}d_m^2 + {}^nC_4c_m^{n-4}d_m^4 - \dots)\\ &\quad +a_m(nc_m^{n-1}d_m - {}^nC_3c_m^{n-3}d_m^3 + {}^nC_5c_m^{n-5}d_m^5 - \dots)\end{aligned}$$

eq. (10) separates into two systems for the real and imaginary parts of such products Simultaneous solution of these two systems then give the unknowns $q_m(\epsilon)$.

3. An Example

Let us solve the third order equations, that is the equations involving q_0, q_1, q_2, and q_3. In this case the system for the a_m's is

$$\left.\begin{aligned} & a_0+a_1+a_2+a_3 = q \\ & c_0a_0+c_1a_1+c_2a_2+c_3a_3 = -q'+b_1d_1+b_2d_2+b_3d_3 \\ & c_0^2a_0+(c_1^2-d_1^2)a_1+(c_2^2-d_2^2)a_2+(c_3^2-d_3^2)a_3 = q'' \\ & \qquad\qquad +2(b_1c_1d_1+b_2c_2d_2+b_3c_3d_3) \\ & c_0^3a_0+(c_1^3-3c_1d_1^2)a_1+(c_2^3-3c_2d_2^2)a_2+(c_3^3-3c_3d_3^2)a_3 = -q''' \\ & \qquad +b_1(3c_1^2d_1-d_1^3)+b_2(2c_2^2d_2-d_2^3)+b_3(3c_3^2d_3-d_3^3) \end{aligned}\right\} \tag{12}$$

while that for the b_m's is obtained from the above with b_m replacing a_m and using $q^{(n)} = 0$ in the right hand side where the rest of the terms are to be multiplied by a negative sign On solving the equations for the b_m's by Cramer's rule, we get,

$$b_1 = D_1/D;\ b_2 = D_2/D,\ b_3 = D_3/D,\quad b_0 = -(b_1+b_2+b_3)$$

Here,

$$\begin{aligned} D &= A_{11}(A_{22}A_{33}-A_{32}A_{23})-A_{12}(A_{21}A_{33}-A_{31}A_{23})+A_{13}(A_{21}A_{32}-A_{31}A_{22}) \\ D_1 &= A(A_{22}A_{33}-A_{32}A_{23})-B(A_{21}A_{33}-A_{31}A_{23})+C(A_{21}A_{32}-A_{31}A_{22}), \\ -D_2 &= A(A_{12}A_{33}-A_{32}A_{13})-B(A_{11}A_{33}-A_{31}A_{13})+C(A_{11}A_{32}-A_{12}A_{31}), \\ D_3 &= A(A_{12}A_{23}-A_{13}A_{22})-B(A_{11}A_{23}-A_{13}A_{21})+C(A_{11}A_{22}-A_{12}A_{21}), \end{aligned}$$

with

$$A = \sum_{m=1}^{3} a_md_m,\quad B = 2\sum_{m=1}^{3} a_mc_md_m,\quad C = \sum_{m=1}^{3} a_md_m(3c_m^2-d_m^2),$$

$$A_{m1} = c_m-c_0,\quad A_{m2} = c_m^2-d_m^2-c_0^2,\quad A_{m3} = c_m^3-3c_md_m^2-c_0^3,$$
$$m = 1, 2, 3, ..$$

Once the b_m's are determined in terms of the a_m's, the a_m's can be obtained as the solution to eq (12) although in a rather laborious fashion. This therefore completes the determination of the constants q_m for the particular order of approximation, and hence the solution of the problem. In all cases, the real and imaginary parts of ξ_m* are obtained as in Sengupta & Srikantiah (1974).

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References

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